AP Review Worksheet #1 Name: Key

Questions from 1998 AB No Calculator Multiple Choice

$y' = x^2 + 10x \quad y'' = 2x + 10$

1. What is the $x$-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

$x = -5$

$y''$: $-6$, $-4$; $-$ | $-5$ | $+$

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) $-5$ (E) $-10$

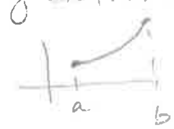
4. If $f$ is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some $c$ such that $a < c < b$.

(B) $f'(c) = 0$ for some $c$ such that $a < c < b$. → There may not be any extrema or PI's on $(a, b)$

horizontal tangent @ c

Counter example:



(C) $f$ has a minimum value on $a \le x \le b$.

(D) $f$ has a maximum value on $a \le x \le b$.

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$ $\frac{-2x - y}{x}$ ⇒ at $x = 2$: $2^2 + 2y = 10$, $y = 3$; $\frac{dy}{dx} = \frac{-2(2) - 3}{2}$

$2x + x\frac{dy}{dx} + (1)y = 0$

(A) $-\frac{7}{2}$ (B) $-2$ (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

8. Let $f$ and $g$ be differentiable functions with the following properties:

(i) $g(x) > 0$ for all $x$

(ii) $f(0) = 1$ ← since $f(x) = \text{constant}$, $f(x) = 1$

$h'(x) = f(x)g'(x) + \underbrace{f'(x)g(x)}_{=0}$

Since $g(x) > 0$, $f'(x) = 0$ ∴ $f(x) = \text{constant}$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) $e^x$ (D) 0 (E) 1

10. What is the instantaneous rate of change at $x = 2$ of the function $f$ given by $f(x) = \frac{x^2 - 2}{x - 1}$?

$f'(x) = [(x-1)(2x) - (x^2 - 2)(1)]/(x-1)^2 \rightarrow f'(x) = \frac{x^2 - 2x + 2}{(x-1)^2} \quad f'(2) = \frac{4 - 4 + 2}{1} = 2$

(A) $-2$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

$\lim_{x \to 2^-} f(x) = \ln 2$, $\lim_{x \to 2^+} f(x) = 4 \ln 2$ — limits $\neq$ so DNE

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

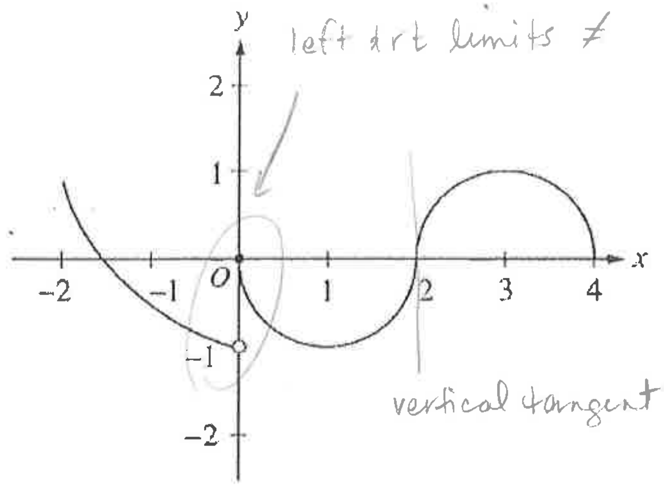
14. A particle moves along the $x$-axis so that its position at time $t$ is given by $x(t) = t^2 - 6t + 5$. For what value of $t$ is the velocity of the particle zero?

$v(t) = 2t - 6$

$0 = 2t - 6$

$t = 3$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

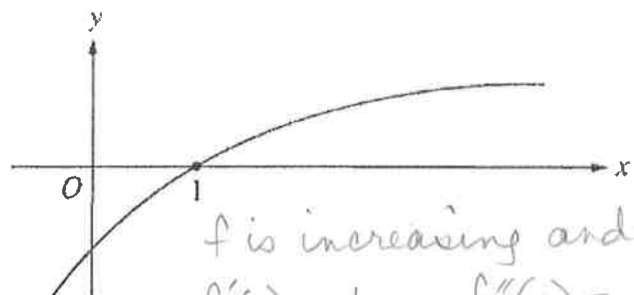



left & rt limits ≠

vertical tangent

13. The graph of the function $f$ shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, −1) and (3, 1). For what values of $x$, $-2 < x < 4$, is $f$ not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3




f is increasing and concave down @ $x=1$

$f'(1) = +$ $f''(1) = -$

17. The graph of a twice-differentiable function $f$ is shown in the figure above. Which of the following is true?

$f''(1) < f(1) < f'(1)$

$-$ $0$ $+$

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

$f''$: −2, −1, −.5, 0, 1, 2, 3; − − +, − + +, + + +, + + +; +, −, +, +

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of $f$ has inflection points when $x =$

(A) −1 only (B) 2 only (C) −1 and 0 only (D) −1 and 2 only (E) −1, 0, and 2 only

24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

$a(t) = 3t^2 - 6t + 12$ $a(0) = 12$ $a(3) = 27 - 18 + 12 = 21$

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

22. The function $f$ is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is $f$ increasing?

$f'(x) = 4x^3 + 2x$

$0 = 2x(2x^2+1)$

$x = 0$

$f'$: $-1$, $0$, $1$; $-\,+$, $+\,+$; $-$, $+$

(A) $\left(-\frac{1}{\sqrt{2}},\ \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$

(C) $(0,\ \infty)$

(D) $(-\infty,\ 0)$

(E) $\left(-\infty,\ -\frac{1}{\sqrt{2}}\right)$

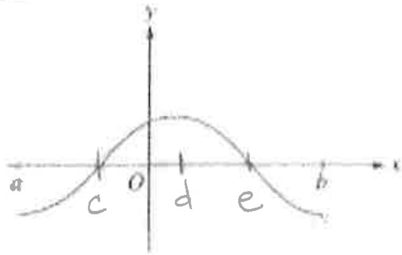
PI @ $x = c$

$f''+$ to $f''-$

slopes increasing

$f'$ inc. to $f'$ decr.

$f'$ max @ $x = c$




$x = a$ $f' = 0$

$x = d$ $f' = 0$

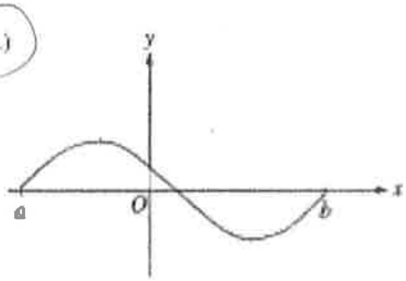
$x = b$ $f' = 0$

$(a,d)$ $f'+$

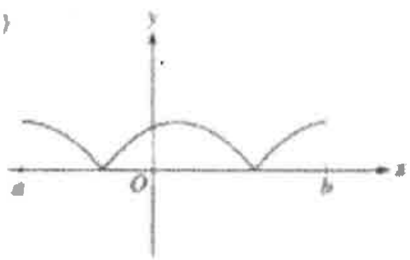
$(d,b)$ $f'-$

23. The graph of $f$ is shown in the figure above. Which of the following could be the graph of the derivative of $f$?

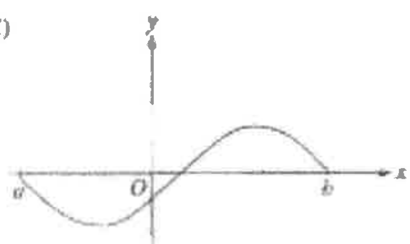
(A)



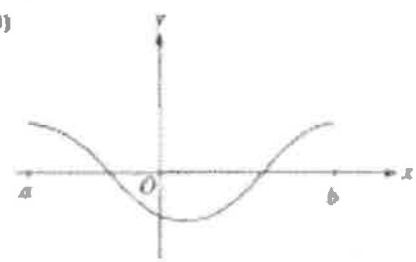
(B)



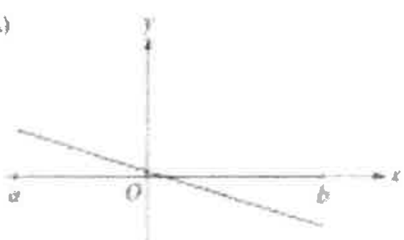
(C)



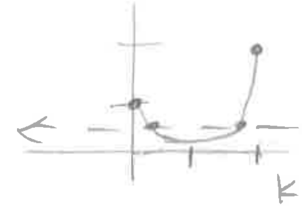
(D)



(E)



| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | $k$ | 2 |




26. The function $f$ is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3